АиС 15.01.13

x^n $\int x^n dx$

кой е израза на която производната му дава x^n

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad \text{при } n$$

$$\Phi \left(\frac{x^{n+1}}{n+1} + const \right)' = x^n ; \int \frac{dx}{x} = \ln x + const$$

$$\int 2x + 1 \, dx = x^2 + x + const$$

(const не е задължително да има)

$df \overset{def}{=} f(x) dx$

$$\int \frac{(x-1)^3}{\sqrt{x}} dx$$ Т. Ньютон

$$\int \frac{x^3 - 3x^2 + 3x - 1}{\sqrt{x}} dx = \int x^{3-1/2} - 3x^{2-\frac{1}{2}} + 3x^{1-\frac{1}{2}} - x^{-\frac{1}{2}} dx$$

Φ $\int af + bg \, dx = a \int f dx + b \int g dx$

линейност

$$\int x^{5/2} dx - 3 \int x^{3/2} dx + 3 \int x^{1/2} dx - \int x^{-1/2} dx$$

от Φ $\int x^n dx = \frac{x^{n+1}}{n+1} \Rightarrow$

$$\frac{2x^{7/2}}{7} - 3\frac{x^{5/2}}{5} \cdot 2 + 3\frac{x^{3/2}}{3} \cdot 2 - 2x^{1/2} + c$$

$$= \frac{2}{7}\left(\sqrt{x}\right)^7 - \frac{6}{5}\left(\sqrt{x}\right)^5 + 2\left(\sqrt{x}\right)^3 - 2\sqrt{x} + c$$

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① $d\,af(x) = a\,df(x)$

$d\,af(x) = (af(x))' dx = af'(x) dx = a\,df(x)$ Φ

$$d f(x) = f'(x)\,dx \;;\; \int f'(x)\,dx = \int d f(x) = f(x) + c$$

$$\int (x-1)^3 \frac{1}{\sqrt{x}}\,dx = \int \left[(\sqrt{x})^2 - 1\right]^3 \frac{1}{\sqrt{x}}\,dx \Rightarrow \frac{1}{\sqrt{x}}dx = d2\sqrt{x}$$

$$\int \left[(\sqrt{x})^2 - 1\right] d2\sqrt{x} \Rightarrow \text{полагаме } t = \sqrt{x}$$

$$d(2\sqrt{x}) \overset{def}{=} (2\sqrt{x})'\,dx = \frac{dx}{\sqrt{x}}$$

$$2\int (t^2-1)^3\,dt = 2\int t^6 - 3t^4 + 3t^2 - 1\,dt =$$

$$= 2\left(\frac{t^7}{7} - 3\frac{t^5}{5} + 3\frac{t^3}{3} - t + c\right)^{t=\sqrt{x}} = \frac{2}{7}(\sqrt{x})^7 + \frac{6}{5}(\sqrt{x})^5 + 2(\sqrt{x})^3 - 2\sqrt{x} + c$$

$$\left((x-2)e^{1/x}\right)' = e^{1/x} + (x-2)e^{1/x} - \left(-\frac{1}{x^2}\right) = e^{1/x}\left[1 + (x-2) + \frac{1}{x^2}\right] =$$

$$= \frac{e^{1/x}}{x^2}\left(x^2 + 2 - x\right)$$

$$\int 2^x + \sqrt{\frac{1}{x}}\,dx = \int 2^x\,dx + \int \sqrt{\frac{1}{x}}\,dx \Rightarrow 2^x \overset{def}{=} e^{x\ln 2} \Rightarrow$$

$$\int e^{x\ln 2}\,dx + \int \frac{dx}{\sqrt{x}}$$

разглеждаме

$$\frac{1}{\ln 2}\int e^{x\ln 2}\,dx\ln 2 \quad \text{или} \quad \frac{1}{\ln 2}\int e^{\ln 2}\ln 2\,dx$$

$$dx\ln 2 = \frac{1}{\ln 2}e^{x\ln 2} + c = \frac{1}{\ln 2}2^x \Rightarrow$$

$$\int e^{x\ln 2}\,dx + \int \frac{dx}{\sqrt{x}} = \frac{2^x}{\ln 2} + 2\sqrt{x} + c$$

$$\int 4\cos x - \frac{5}{\sqrt{9-9x^2}}\,dx = 4\int \cos x\,dx - 5\int \frac{dx}{\sqrt{9-9x^2}} =$$

$$= 4\int \cos x\,dx - \frac{5}{3}\int \frac{dx}{\sqrt{1-1x^2}} = 4\sin x - \frac{5}{3}\arcsin x + c$$

$$\int (2x-3)^{10}\,dx \quad \Phi\, d(f(x)+c) = (f(x)+c)'dx = f'(x)dx = df(x)$$

$$\int (2x-3)^{10} d\,\frac{2x}{2} = \int (2x-3)^{10}\frac{1}{2}\,d2x = \frac{1}{2}\int (2x-3)^{10}\,d2x =$$

$$= \frac{1}{2}\,\frac{(2x-3)^{11}}{11} + c \qquad \left(\int x^{10}dx = \frac{x^{10+1}}{10+1} = \frac{x^{11}}{11}\right)$$

$$\int \frac{dx}{1-\sin x} = \int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}} = \int \frac{dx}{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2} =$$

$$\int \frac{dx}{\cos^2\frac{x}{2}\left(\operatorname{tg}\frac{x}{2}-1\right)^2} = \left(\frac{1}{\cos^2\frac{x}{2}}\right)\frac{1}{\left(\operatorname{tg}\frac{x}{2}-1\right)^2}\,dx \Rightarrow (\operatorname{tg}x)' = \frac{1}{\cos^2 x}\ =$$

$$2\int \frac{dx}{\cos^2\frac{x}{2}} = 2\operatorname{tg}\frac{x}{2} \Rightarrow \int \frac{d\,2\operatorname{tg}\frac{x}{2}}{\left(\operatorname{tg}\frac{x}{2}-1\right)^2} =$$

$$= 2\int \frac{d\left(\operatorname{tg}\frac{x}{2}-1\right)}{\left(\operatorname{tg}\frac{x}{2}-1\right)^2} \Rightarrow \int \frac{dx}{x^2} = -\frac{1}{x} \Rightarrow$$

$$-2\,\frac{1}{\operatorname{tg}\frac{x}{2}-1} + c$$